DEFENCE RESEARCH ESTABLISHMENT PACIFIC
VICTORIA, B.C.

This information is furnished with the express understanding that
(a) Proprietary and patent rights will be protected.
(b) It will not be released to another nation without specific approval of the Canadian Department of National Defence.

DREP Report 71 - 3

PREDICTIONS OF TEMPERATURES IN SNOW-FREE SEA ICE WITH HOURLY CHANGES IN ATMOSPHERIC HEAT FLUXES

by
A.R. Milne

October 1971

DEFENCE RESEARCH BOARD
CANADA

DEFENCE RESEARCH ESTABLISHMENT PACIFIC

VICTORIA, B.C.

DREP Report 71-3

PREDICTIONS OF TEMPERATURES IN SNOW-FREE SEA ICE WITH HOURLY CHANGES IN ATMOSPHERIC HEAT FLUXES

by

A.R. Milne

October 1971

Approved for distribution

Director General

DEFENCE RESEARCH BOARD

CANADA

ABSTRACT

Calculations are made of temperatures vs depth and time in snow-free winter ice with hourly atmospheric heat fluxes as input data for the month of April 1968. Heat fluxes were derived from weather observations made at stations located in the Canadian Arctic and are assumed to be valid for the ice cover in western Parry Channel. A simplified analytical model is justified on the basis that the prime interest is in temperature fluctuations of a daily period near the ice-air boundary. Results are shown for different assumed ice surface albedos and short wave extinction coefficients.

INTRODUCTION

The need to predict short-term temperature changes vs time in a sea ice layer arose out of the observation that surface tension cracks produced underwater noise and that the noise responded to decreases in the air temperature (Ganton and Milne, 1965). Of particular interest were the variations in ice temperatures vs time that existed during the period 16 April to 27 April, 1968. During this period fluctuations in the air temperature corresponded to antiphase fluctuations in the underwater noise envelope recorded in M'Clure Strait. These fluctuations are shown in figure 1 and M'Clure Strait is shown in the map of figure 2.

The calculation method used was to assume snow free winter ice of a given thickness and to synthesize the smooth interchange of heat fluxes with time between the ice and the atmosphere by the use of hourly step function changes in heating or cooling. By summing solutions of the equation of heat conduction appropriate to these step function changes in heat flux, temperatures vs depth and time were obtained.

Maykut and Untersteiner (1969) perform much more detailed calculations of a similar nature which were designed to predict the equilibrium conditions of the growth and decay of sea ice. Their input data was primarily mean monthly heat fluxes. Thus, given initial conditions of ice and snow thickness, the effect of various parameters could be explored with respect to the equilibrium conditions achieved after the passage of several years. In contrast, the calculations in this report use hourly heat fluxes as input data, the intent being to explore the maximum temperature changes (snow free) vs time that are likely in a sea ice layer with respect to variations in parameters such as the albedo and extinction coeeficient. Hourly heat fluxes, used for input data, were derived from nearby Arctic weather station observations - primarily from Mould Bay,

for the period of interest (figure 2).

The prediction of transient temperature changes in a floating sheet of sea ice is a prerequisite in the estimation of variations in lateral stresses and strains with depth. The compressive and tensile stresses which result from temperature changes are important in the consideration of ice pressures exerted on structures and in the occurrence of thermal fissures or cracks such as those described by Zubov (1943) and Kingery et al.(1963). Thermal tension cracks are important sources of impulsive underwater noises. Milne and Ganton (1971) summarize the characteristics of this type of under ice noise.

It is difficult to predict the actual lateral stresses and strains induced by temperature changes in an ice sheet because of the general lack of knowledge of the macro-viscoelastic properties of sea ice. Tabata (1948,1967) shows that sea ice which is warm and saline is more plastic than when cold and less saline, and Voitkovsky (1967), in his measurements of the relaxation of stresses in fresh ice, shows that ice is a non-linear viscoelastic solid. The result is that sea ice has viscoelastic properties which depend on its temperature, salinity and stress history, all of which are unknown in detail. To complicate matters further measurements of salinities vs depth in winter ice of apparently uniform thickness vary greatly depending on the location as shown by Bennington (1967).

In view of the above uncertainties it seems reasonable to make some simplifying assumptions in the calculation of temperatures vs time in a layer of sea ice. The most basic of these is to assume that the thermal conductivity and diffusivity are constants, independent of salinity

and temperature. Other simplifying assumptions include a constant albedo at the ice-air interface and a constant extinction coefficient for the solar radiation which penetrates the ice and causes internal heating. Constant values for the conductivity , diffusivity and the extinction coefficient imply that the ice layer is thermally homogeneous so that the one-dimensional diffusion equation can be used, with boundary conditions that need only be applied at the ice-air and the ice-water interface.

THE HEAT CONDUCTION PROBLEM

Solutions of the equation of heat conduction

Figure 3 shows the geometry of the one-dimensional heat conduction problem, where z = H is the ice thickness and z = 0 defines the ice-air boundary. The net heat flux, positive in the +z direction, is shown as QS, and is in units of cal $cm^{-2}hr^{-1}$. It is assumed that the net heat flux, QS, enters the ice layer at z = 0, some of which heats the boundary layer at z = 0, while a radiative component is dissipated throughout the ice layer and the water beneath. The water is assumed to be a constant temperature bath.

From p.29 of Carslaw and Jaeger (1960), the appropriate one-dimensional equation of heat conduction is:

$$\frac{\partial^2 T}{\partial z^2} - \frac{1}{a^2}\frac{\partial T}{\partial t} = -\frac{A}{k}, \qquad 0 \le z \le H \tag{1}$$

where T = T(z,t) is the temperature at depth z and time t,

k is the thermal conductivity,

$a^2 = \frac{k}{\rho c}$ is the thermal diffusivity, with

ρ the density of sea ice,

c the specific heat;

and A is the rate of heat production per unit depth.

Since a net heat flux, QS, is assumed to have entered the ice at the plane $z = 0$, the boundary conditions are:

(1) no heat conduction exists at $z = 0$ ($\partial T/\partial z = 0$ at $z = 0$),

and (2) at $z = H$, $T = 0$.

In a homogeneous solid subjected to radiant heating, the rate of heat production per unit depth decays exponentially so that:

$$A = A_o \exp(-Kz) \tag{2}$$

where K is an "extinction coefficient". If a part of the total heat flux, QS, which causes internal heating, is termed, Q, then

$$Q = A_o \int_o^\infty \exp(-Kz)dz = A_o/k. \tag{3}$$

(Later, it is expedient to find a solution for heating which is concentrated near $z = 0$ by letting the extinction coefficient, K, approach infinity).

Using (2) and (3), equation (1) becomes:

$$\frac{\partial^2 T}{\partial z^2} - \frac{1}{a^2}\frac{\partial T}{\partial t} = -KQ\exp(-Kz)/k, \qquad 0 \leq z \leq H \tag{4}$$

The total heat flux, QS, normally has several components, all of which can be assigned different extinction coefficients so that if $QS = Q_1 + Q_3 + \ldots Q_n$, then (4) can be written as the set of equations:

$$\frac{\partial^2 T_1}{\partial z^2} - \frac{1}{a^2}\frac{\partial T_1}{\partial t} = -K_1 Q_1 \exp(-K_1 z)/k$$

$$\frac{\partial^2 T_2}{\partial z^2} - \frac{1}{a^2}\frac{\partial T_2}{\partial t} = -K_2 Q_2 \exp(-K_2 z)/k$$

$$- - - - - - \qquad - - - - - - - - -$$

$$\frac{\partial^2 T_n}{\partial z^2} - \frac{1}{a^2}\frac{\partial T_n}{\partial t} = -K_n Q_n \exp(-K_n z)/k \tag{5}$$

where $T = T_1+T_2+...T_n$.

If all of one component of heat flux, say Q_n, is totally absorbed near z= 0, then T_n in (5), with the extinction coefficient K_n set to infinity, is also a solution.

Considering the current problem, the total heat flux, QS, can be divided into two parts, one part Q_I which produces internal heating and a second part Q_V which produces surface heating at z = 0.

The required solution to (4) for the step function of heat flux $Q(t) = Q_I$, $t > 0$ only, was obtained using the method on p. 130 of Carslaw and Jaeger (1960), where the temperature T(z,t) in $0 \leq z \leq H$ is expressed as the sum of a steady state and a time dependent solution of (4) as follows:

$$T(z,t) = u(z) + \omega(z,t), \tag{6}$$

where

$$\frac{\partial^2 u}{\partial z} = -KQ_I\exp(-Kz)/k, \quad u = u(z) \text{ only.}$$

Hence $u(z) = -Q_I\exp(-Kz)/(kK) + C_1z + C_2$, C_1 and C_2 constants. Applying boundary conditions $\frac{\partial u}{\partial z} = 0$ at $z = 0$ and $u = 0$ at $z = H$,

$$u(z) = (Q_I/k)\left\{(H-z) - \exp(-kH)\left[\exp(-K(H-z))-1\right]/K\right\} \tag{7}$$

From (6) and (7), equation (4) now becomes:

$$\frac{\partial^2\omega}{\partial z^2} - \frac{1}{a^2}\frac{\partial\omega}{\partial t} = 0, \quad 0 \leq z \leq H \tag{8}$$

For the boundary conditions $\frac{\partial\omega}{\partial z} = 0$ at $z = 0$, $\omega = 0$ at $z = H$, a series solution to (8) is:

$$\omega(z,t) = \Sigma_{n=o}^{\infty}\left\{b_n\exp(-a^2(2n+1)^2\pi^2t/(2H)^2)\cos((2n+1)\pi z/(2H))\right\}, \tag{9a}$$

with $$\omega(z,o) = \sum_{n=o}^{\infty} \left\{ b_n \cos((2n+1)\pi(2H)) \right\} . \tag{9b}$$

Since no heat is applied until $t > 0$, then from (6)

$$u(z) + \omega(z,o) = 0,$$

and from (7),

$$\omega(z,o) = (Q_I/k) \left\{ (H-z) - \exp(-KH) \left[\exp(-K(H-z)) - 1 \right] /K \right\} \tag{10}$$

The coefficients, b_n, in the sum (9b) are given by:

$$b_n = (2/H) \int_o^H \omega(z,o) \cos((2n+1)\pi z/(2H))dz. \tag{11}$$

Substituting (10) in (11), and integrating:

$$b_n = - \frac{4Q_I}{(2n+1)kK\pi} \left\{ (-1)^n \exp(-KH) + \frac{2HK}{\pi(2n+1)} \right\} \left\{ 1 - \left(1 + \left[\frac{2HK}{\pi(2n+1)} \right]^2 \right)^{-1} \right\} \tag{12}$$

From (6), (7), (9a) and (12), the solution to (4) for a step function of heating flux $Q(t) = Q_I$, $t > 0$ only, is termed $T_I(z,t)$, where the subscript "I" refers to internal heating, and is given by:

$$T_I(z,t) = \frac{Q_I}{kK} \left\{ \exp(-KH) - \exp(-Kz) + K(H-z) \right\} - \frac{4}{\pi} \frac{Q_I}{kK} \sum_{n=o}^{\infty} \left\{ (2n+1)^{-1} \left[(-1)^n \exp(-KH) + \frac{2HK}{\pi(2n+1)} \right] \right.$$

$$\times \left[1 - \left(1 + \left[2HK/(\pi(2n+1)) \right]^2 \right)^{-1} \right] \exp\left[-a^2 (2n+1)^2 \pi^2 t/(2H)^2 \right]$$

$$\left. \times \cos\left[(2n+1)\pi z/(2H) \right] \right\} \tag{13}$$

For the case where all the heat flux causes heating solely in the vicinity of $z = o$; a second solution, termed $T_U(z,t)$, where the subscript "U" refers to upper surface heating, is obtained from (13) by letting the extinction coefficient, K, approach infinity; hence:

$$T_U(z,t) = \frac{Q_U}{k}(H-z) - \frac{8HQ_U}{k\pi^2} \sum_{n=o}^{\infty} \left\{ (2n+1)^{-2} \exp \left[-a^2(2n+1)^2\pi^2 t/(2H)^2 \right] \right.$$

$$\left. \times \cos \left[(2n+1)\pi z/(2H) \right] \right\} . \qquad (14)$$

Figures (4a) and (4b) are plots of $T_I(z,t)$ from (13) and figure (4c) shows plots of $T_U(z,t)$ from (14). The curves in each case are for constant times, t, and the families are for 20 hr increments in time beginning at t = 0.5 hrs. In figure (4a), Q_I = 1.75 cal $cm^{-2}hr^{-1}$ and K is 0.0170 cm^{-1}. In figure (4b), Q_I = 1.75 cal $cm^{-2}hr^{-1}$ but the extinction coefficient K is decreased to 0.0085 cm^{-1}. Figure (4c) shows the effect of K equal to infinity (surface heating only) and where Q_U = 0.875 cal $cm^{-2}hr^{-1}$. The other constants used in (13) and (14) were as follows: ice thickness, H = 200 cms; conductivity k = 17.5 cal cm^{-1} $^oC^{-1}$ hr^{-1}, and the diffusivity a^2 = 30.2 cm^2hr^{-1}.

Heat Fluxes

Walmsley (1966) describes in detail the ice cover and surface heat fluxes in Baffin Bay and in particular the interelationship of surface weather observations with computed heat fluxes. His choice of expressions, describing the convective and latent heat fluxes,is used herein.

The heat flux balance can be expressed as follows:

$$QR + QE + QC = QS, \qquad (15)$$

where QR is the net radiation flux,

QE is the latent heat flux brought about by the evaporation or condensation of water substance,

QC is the sensible or convective heat flux,

and QS is the net heat flux change stored in the sea ice and the ocean beneath.

The convention adopted is that all heat fluxes are positive in the +z direction (see figure 3), i.e., when heat is added to the ice.

Convective heat flux, QC

Walmsley (1966) uses Shuleikin's (1953) formula for QC, where:

$$QC = 1.27\ (T_a - T)\ \text{cal cm}^{-2}\text{hr}^{-1},\ T \geq T \qquad (16a)$$

$$\text{or } QC = .0175\ (T_a - T)V\ \text{cal cm}^{-2}\text{hr}^{-1},\ T < T_a, \qquad (16b)$$

in (16) T_a is the air temperature (^{o}C) at a height of 8m, T is the surface temperature (^{o}C) of the ice and V is the wind speed at 8 m height (m sec^{-1})

Latent heat flux, QE

Sverdrup's (1951) formula is used by Walmsley, where the evaporation of water from a sea surface is given by:

$$E = \kappa(W_i - W_a)V\ \text{cm day}^{-1},$$

where W_i, W_a are water vapour pressures at the ice surface and in the air,

V is the wind speed and

κ is a coefficient.

The latent heat flux is given by

$$QE = \alpha LE$$

where α is the density of the ice,

L is the sum of the latent heats of fusion and sublimation and

E is the evaporation.

The water vapour pressures W_i and W_a are expressed in terms of the ice surface temperature, T(^{o}C) and the dew point T_d(^{o}C) at a height of 8m, and after establishing suitable values for the coefficient κ, Walmsley uses the following expressions:

$$QE = 2.26\left[\exp(.0728T_d) - \exp(.082T)\right] V\ \text{cal cm}^{-2}\text{hr}^{-1} \qquad (17a)$$

$$\text{for } V > 6.2\,\text{m sec}^{-1};$$

$$1.4\left[\exp(.0728T_d) - \exp(.082T)\right] V\ \text{cal cm}^{-2}\text{hr}^{-1}, \qquad (17b)$$

$$\text{for } V \leq 6.2\,\text{m sec}^{-1}.$$

It can be seen that the heat fluxes QC in (16) and QE in (17) are sensitive to the ice surface temperature which is to be calculated. This difficulty is partly resolved by assigning the ice surface temperature calculated for the previous hour to the computation of QC and QE for the current hour. Both fluxes QC and QE interact at the ice-air boundary ($z = 0$ in figure 3).

Radiative heat fluxes, QR

Following the notation of Schwerdtfeger and Pounder (1963), the net heat flux from electromagnetic radiation is given by:

$$QR = A_i - A_o,$$

where A_i and A_o are the total incident and outgoing radiative fluxes for all wavelengths. A_i and A_o are separated into the short wave (0.2 to 4 micron) fluxes S_i and S_o and the long wave (3 to 80 micron) fluxes L_i and L_o, so that $A_i = S_i+L_i$ and $A_o = S_o+L_o$. The long wave radiation is entirely absorbed and emitted in the vicinity of the ice-air boundary ($z = 0$), so that L_o can be expressed as $L_o = \varepsilon_L \sigma T_K^4$, where ε_L is the long wave emissivity, σ is the Stefan-Bobtzmann constant and T_K is the ice surface temperature (^oK). The short wave radiation, on the other hand is partly reflected from and partly transmitted through the ice surface according to its albedo, α. Therefore $S_o = \alpha S_i$ and the short wave net heat flux is $(1-\alpha)S_i$. A fraction of $(1-\alpha)S_i$ causes internal heating which changes with depth according to an "average" extinction coefficient, K, for the short wave radiation band.

Maykut and Untersteiner (1969) introduce an internal heating flux I_o, where the difference between $(1-\alpha)S_i$ and I_o is a contributor to surface heating. It is difficult to estimate the fraction $I_o/((1-\alpha)S_i)$ since it is likely to range from near unity for a low albedo to .1 or so

when the ice surface has a high albedo. This is equivalent to saying that there can be a thin layer near $z = 0$ which has a high extinction coefficient and at the same time reflects most of the short wave radiation. For the purposes of the calculations in this report, $I_o = .5(1-\alpha)S_i$. To sum up,

$$QR = L_i - L_o + (1-\alpha)S_i \qquad (18)$$

where the flux $L_i - L_o + (1-\alpha)S_i - I_o$ causes surface heating, and the flux I_o causes internal heating.

The Storage heat flux, QS

QS is the sum of all three heat fluxes QR, QC and QE and appears as heat loss or gain in the ice and the ocean beneath. A small fraction of the short wave radiation is lost to the water, which is assumed to be an infinite energy sink. The balance is stored in:(1) heat changes in the ice layer, (2) changes in the latent heat of fusion at the ice-water interface, and (3) in the convection of heat from the water. Lewis(1967) indicates that 10% of this balance, under conditions of ice growth, comes from the water, while under equilibrium conditions, the remainder is devoted to the accretion of ice so that the latent heat of fusion is the primary sink of heat.

For the purposes of the problem described in this report, the fraction of the short wave radiation energy lost to the water is ignored and it is assumed that 90% of the daily sum of QS, if negative, is devoted to the liberation of the latent heat of fusion in the accretion of ice. For each day, this accretion, in cms, is added to the initial thickness. The daily accretion is therefore assumed to be

$$\Delta H = -\frac{.9}{L\rho}\langle QS\rangle \text{cm day}^{-1} \qquad (19)$$

where $\langle QS\rangle$ is the daily average of (QC+QE+QR) in units of cal $cm^{-2}hr^{-1}$. The value for the latent heat of fusion, L, and the density of sea ice, ρ, at the growing interface were taken from Lewis (1967), where

$L = 67.5$ cal gm^{-1} and $\rho = .915$ gm cm^{-3}. Using these values, (19) becomes:

$$\Delta H = - .35 \langle QS \rangle \text{ cm day}^{-1}.$$

Heat flux data inputs

Since no heat flux measurements were made over the ice cover of interest, reliance was placed on the weather observations made primarily at Mould Bay and at Resolute Bay. The intention was not to replicate true temperatures vs time in the ice sheet but to obtain typical temperatures which would be produced by similar weather systems during the month of April. Air temperatures, T_a, dew points T_d, wind speeds, V, and their directions were available from the bi-yearly "Arctic Summary" publications* for Mould Bay. These values are listed every 4 hours. For the purposes of calculation, hourly values were obtained by a linear interpolation procedure. Radiation observations were obtained from the "Monthly Radiation Summary" publications.* These are listed hourly, however not all necessary observations were obtainable from Mould Bay so that some of the missing radiation observations were obtained from Resolute Bay.

The calculation of QC from (16) and QE from (17) is therefore relatively straight forward; however the calculation of QR in (18) from the available radiation information is somewhat more complex. The available radiation data of use was:

RF1, the global solar radiation, downward and diffuse,

RF3, the reflected solar radiation, upward and reflected,

RF4, the net radiation that is the net flux of downward and upward total solar, terrestrial and atmospheric radiation.

Therefore, at the weather station,

$$S_i - S_o + (L_i - L_o) = RF1 - RF3 + (L_i - \varepsilon_L \sigma T_K^4) = RF4,$$

from which $(L_i - L_o) = RF4 + RF3 - RF1.$ (20)

* Available from the Department of Transport, Meteorological Branch, Canada.

Recalling that $L_o = \varepsilon_L \alpha T_K^4 = \varepsilon_L \alpha (T+273)^4$, there was no other choice available than to assume that surface temperature T at the weather station approximated T at the ice-air interface insofar as the long wave radiation was concerned. For the short wave radiation, RF1 = S_i, so that over the ice

$$S_i - S_o = (1-\alpha)S_i = (1-\alpha)\mathrm{RF1}, \qquad (21)$$

where α is the albedo of the sea ice. Therefore from (20) and (21), (18) can be written as:

$$\mathrm{QR} = \mathrm{RF4} + \mathrm{RF3} - \alpha(\mathrm{RF1}) \quad \mathrm{cal\ cm^{-2}hr^{-1}}, \qquad (22)$$

of which

$$\mathrm{RF4} + \mathrm{RF3} - \alpha(\mathrm{RF1}) - I_o \qquad (23)$$

causes surface heating, and

$$I_o = 0.5(1-\alpha)\mathrm{RF1} \qquad (24)$$

causes internal heating. I_o is hereafter given the Fortran name 'RADQ'.

The net heat flux which caused surface heating in the vicinity of z = 0 is given the Fortran name 'SURFQ' and is the sum of QC, QE and the radiation surface heating from (23):

$$\mathrm{SURFQ} = \mathrm{QR} + \mathrm{QC} + \mathrm{QE} - \mathrm{RADQ}. \qquad (25)$$

As mentioned previously, values of QC and QE depended on the calculated surface-temperatures of the sea ice and were made available only for the hour that had passed. The radiation terms, air temperatures and dew points were stored in advance.

Numerical procedure

At any hour, the temperature vs depth depended on the past history of heat flux inputs, and since the solutions $T_I(z,t)$ in (13) and $T_S(z,t)$ in (14) are applicable to step function heat flux inputs, then the temperature T at hour N was a result of a set of heat flux steps:

(RADQ (N-m) - RADQ (N-m-1)), from (24) and

(SURFQ (N-m) - SURFQ (N-m-1)), from (25),

where $m = 0,1,2...N-2$.

The temperature vs depth at hour N can be expressed as follows:

$$T(z,N) = T(z,1) + \Sigma_{m=0}^{N-2} \; T_S(z,m+\tfrac{1}{2}) + T_I(z,m+\tfrac{1}{2}) \quad , \; (26)$$

with $N = 2,3...NMAX$,

Q_U in (14) set equal to equal to SURFQ(N-m) - SURFQ(N-m-1),

and Q_I in (13) set equal to equal to RADQ(N-m) - RADQ(N-m-1).

T(z,1) is a linear temperature vs depth profile assumed as an initial condition. In (26) $T(z,1) = (-1.56+H\langle QS(1)\rangle/k)$ ^{o}C, where $\langle QS(1)\rangle$ is the daily average of QC+QE+QR in cal $cm^{-2}hr^{-1}$ for the first day and the sea water temperature is -1.56 ^{o}C.

Constants used in the calculations

The thermal conductivity, k, and the diffusivity, $a^2 = k/(\rho c)$ are variables in sea ice which depend on the temperature and salinity variation with depth. Untersteiner's (1961) formulae express this variability as follows:

$$k = 17.5 + 1.01\; S(z)/T(z) \text{ cal cm}^{-1}\, {}^{o}C^{-1}\, hr^{-1},$$

and

$$a^2 = k/(\rho c) = k/(0.45+4.1\; S(z)/T^2(z)) \; cm^2hr^{-1},$$

where S(z) is the salinity in parts per thousand (‰) and T(z) is the temperature in ^{o}C.

Figure 5 b shows k and a^2 vs depth for the winter ice conditions shown in figure 5a and figure 5d shows k and a^2 vs depth for the polar ice conditions shown in figure 5c. The diffusivity is considerably more variable with temperature, as a result of the changes in the specific heat c. The higher specific heat is a result of the storage of heat in the changing volumes of

brine, which increases in volume nearer the melting point. A good description of this phenomenon is given by Schwerdtfeger (1962).

The consequences of assuming that the conductivity, k, is a constant are relatively insignificant. The consequences of assuming a constant diffusivity, a^2, need, however, to be assessed in the light of the problem under consideration. For the steady state, a^2 does not influence the temperature profile. When the heat fluxes vary with time, it can be seen from an examination of equations (13) and (14) that decreasing values of a^2 require increasing times t to obtain the identical temperatures, and therefore reflect the increasing heat energy storage capability of warmer sea ice. Consequently, an assumption of a constant value of a^2, say 30.2, to represent reality for the upper 100 cm of winter ice (figure 5b) will predict transient temperature fluctuations which are increasingly too large as the ice-water boundary is approached. Because short period temperature fluctuations are rapidly damped with increasing depth, errors will only be significant with regard to longer period temperature fluctuations. Even these errors are likely to be irrelevent to the calculation of lateral tensile or compressive stresses because of the increased plasticity of sea ice with increases in salinity and temperature.

For the purposes of the current problem, $k = 17.5$ cal $cm^{-1}\ {}^{o}C^{-1} hr^{-1}$ and $a^2 = 30.2\ cm^2 hr^{-1}$.

Two other constants required are the albedo, α, and the extinction coefficient, K cm^{-1}. The albedo is the ratio of the reflected to incident radiation and the extinction coefficient defines the decay in the intensity of the solar radiation flux with depth. Both α and K

depend on the electromagnetic wavelength. Measurements of the albedo, α, are described by Langleben (1969), Briazgin (1959) and Maykut and Untersteiner (1969). From these authors, a value of α = .65 was chosen to be representative of the albedo of snow free winter ice in April. This value is likely to be, on the average, somewhat low, however the error is in the direction which would lead to higher surface temperature fluctuations. The extinction coefficient for the solar radiation penetration of winter ice does not appear to be specifically available in the literature. For sea ice, unspecified values of K are given by Thomas (1963) and Roulet (1970) where the latter author shows the variability in K with depth and electromagnetic wavelength. A value of K = .0170 was chosen to be representative of winter ice in April.

RESULTS

Figures 6a, 6b and 6c are results of calculations for the period 2300 hrs (LMT) 15 April to 2300 hrs (LMT) 27 April, 1968, inclusive. (Calculations actually commenced at 2300 hrs (LMT) on the 11th of April in order to diminish the effect of an incorrect thermal history). Figure 6a shows predictions of ice temperature (^{o}C) vs time at depths of 0.0, 8.0 and 16.0 cms. Features shown are (1) a steady increase in the average temperature, (2) decreased amplitude and a phase shift with increased depth for the temperature fluctuations with a daily period, and (3) the low pass filtering effect of the ice on the higher frequency temperature fluctuations. Constants chosen were: conductivity k = 17.5 cal $cm^{-1}\ {}^{o}C^{-1}hr^{-1}$, diffusivity a^2 = 30.2 cm^2hr^{-1}, extinction coefficient K = 0.017 cm^{-1}, albedo α = 0.65. As in equation (24), one-half the short wave radiation flux was assumed to be absorbed at the ice-air boundary and the remainder internally. The initial thickness of the ice H was assumed to be 213 cm on the 12th of April.

Figure 6b shows the heat fluxes QR,QE and QC from (16), (17) and (22) in cal $cm^{-2}hr^{-1}$. It can be seen that the latent heat flux, QE, is very small and that the radiative heat flux, QR, was dominant during the period 16 to 27 April, 1968. The maximum values of QR were near 1400 hrs, local mean time, where the ice cover heat gain was a maximum. The convective heat flux, QC, remained negative, indicating that the ice surface temperature remained warmer than the air temperature according to (16a). In fact, QC remained roughly in phase with QR indicating that radiation cooling from the bare ice surface at night resulted in ice surface temperature decreases less in magnitude than air temperature decreases. Whether or not such a situation was realistic could only have been revealed if air temperatures over the ice had been available, rather than the air temperatures at Mould Bay.

Figure 6c shows the internal heating flux, RADQ, from (24) and the surface heating flux, SURFQ, from (25) which were used in (26) to compute the temperature vs time in figure 6a. Daily variations in RADQ are a result of the presence and absence of solar radiation entirely.

Calculations shown in Figures 7 and 8 are similar to figure 6 except for changes in the albedo, α, and extinction coefficient, K, all other constants remaining the same. In figure 6 the calculations were for $\alpha = 0.65$ and $K = 0.017\ cm^{-1}$, while in figure 7 for $\alpha = 0.75$ and $K = 0.034\ cm^{-1}$, and in figure 8 for $\alpha = 0.55$ and $K = 0.0085\ cm^{-1}$. It was assumed that if the albedo was lower, the ice was likely to be more transparent, so that a lower extinction coefficient should be used; conversely, a higher albedo was assumed to be accompanied by a higher extinction coefficient.

It is interesting to observe that the calculated amplitude of the daily fluctuations in ice temperature varied little between figures 7a

and 8a; although for the higher albedo in figure 7, the slowly rising trend in surface temperature was about $3^{\circ}C$ lower as a result of more reflection of solar radiation. The daily periodic fluctuations in ice temperatures in figure 7 for depths of 0.0, 8.0 and 16.0 cms tended to remain similar to those in figure 8 as a result of the higher extinction coefficient used.

ACKNOWLEDGEMENT

Mr. F.G. Greensmith provided indispensable support in data sorting and the preparation of visual material.

REFERENCES

1. Bennington, K.O., "Desalination Features in Natural Sea Ice", J. of Glaciology, Vol. 6, No. 48, pp. 485-857, 1967.

2. Briazgin, N.N., "The Problem of the Albedo of the Surface of Drifting Ice", in Problemy Arktiki i Antarktiki, No. 1, pp. 33-39, 1959.

3. Carslaw, H.S., and J.C. Jaeger, "Conduction of Heat in Solids", Oxford University Press, 1960.

4. Ganton, J.H., and A.R. Milne, "Temperature and Wind Dependent Ambient Noise under Midwinter Pack Ice", J. Acoust. Soc. Am., Vol. 38, No. 3, 406-411, Sept. 1965.

5. Kingery, W.D., and R.L. Coble, "Cracks in Sea Ice and Their Effect on Operations", in "Ice and Snow", edited by W.D. Kingery, M.I.T. Press 1963.

6. Langleben, M.P., "Albedo and Degree of Puddling of a Melting Cover of Sea Ice", J. of Glaciology, Vol. 8, No. 54, pp. 407-412 October 1969.

7. Maykut, G.A., and N. Untersteiner, "Numerical Prediction of the Thermodynamic Response of Arctic Sea Ice to Environmental Changes", Rand Corp. Memorandum RM-6093-PR, 1700 Main St., Santa Monica, California 90406, USA, November 1969.

8. Lewis, E.L., "Heat Flow Through Winter Ice", in "Physics of Snow and Ice, Vol I, part 1", pp. 611-631, Inst. Low Temp Sci., Hokkaido University, Japan, 1967.

9. Milne, A.R., and J.H. Ganton, "Noise Beneath Sea Ice and its Dependence on Environmental Mechanisms", U.S.N. J. Underwater Acoustics, Vol. 21, No. 1, 69-81, January 1971.

10. Roulet, R.R., "A Portable Spectrophotometer for Geophysical Applications", Sci. Report of the Dept. Atmospheric Sci., University of Washington, December 1970.

11. Schwerdtfeger, P., and E.R. Pounder, "Energy Exchange Through an Annual Sea Ice Cover", International Assoc. of Sci. Hydrology, Commission on Snow and Ice, pp. 109-119, Assemblée Générale de Berkeley, California, 19-31 August 1963.

12. Schwerdtfeger, P., "Thermal Properties of Sea Ice", in Report S-10, Ice Research Project, McGill University, Montreal, December 1962.

13. Shuleikin, V.V., "Molecular Physics of the Sea", in Physics of the Sea, Part VIII, pp. 727-786, (translated from the Russian by the U.S. Hydrographic Office, 1957), 1953.

14. Sverdrup, H.U., "Evaporation from the Oceans", in "Compendium of Meteorology", pp. 1071-1081, Edited by T.F. Malone, American Meteor. Soc., 1951.

15. Tabata, T., "Studies of the Mechanical Properties of Sea Ice X. The Flexural Strength of Small Sea Ice Beams", in "Physics of Snow and Ice", Vol. I, Part 1, pp. 481-497, Inst. of Low Temp. Sci., Hokkaido University, Japan, 1967.

16. Tabata, T., "Studies on the Viscoelastic Properties of Sea Ice", in "Arctic Sea Ice", pp. 139-147, U.S. Nat. Acad. Sci.-Nat. Res. Council pub. #598, 1958.

17. Thomas, C.W., "On the Transfer of Visible Radiation through Sea Ice and Snow", J. of Glaciology, Vol. 4, pp. 481-484, February 1963.

18. Voitkovsky, K.F., "The Relaxation of Stresses in Ice", in "Physics of Snow and Ice", Vol. I, part 1, pp. 329-337, Inst. of Low Temp Sci., Hokkaido University, Japan, 1967.

19. Walmsley, J.L., "Ice Cover and Surface Heat Fluxes in Baffin Bay", Marine Sciences Centre, McGill University, Montreal, Manuscript Report No. 2, October 1966.

20. Zubov, N.N., "Arctic Ice", translated from the Russian by the U.S.N Oceanographic Office and the American Meteorological Society, original: 1943.

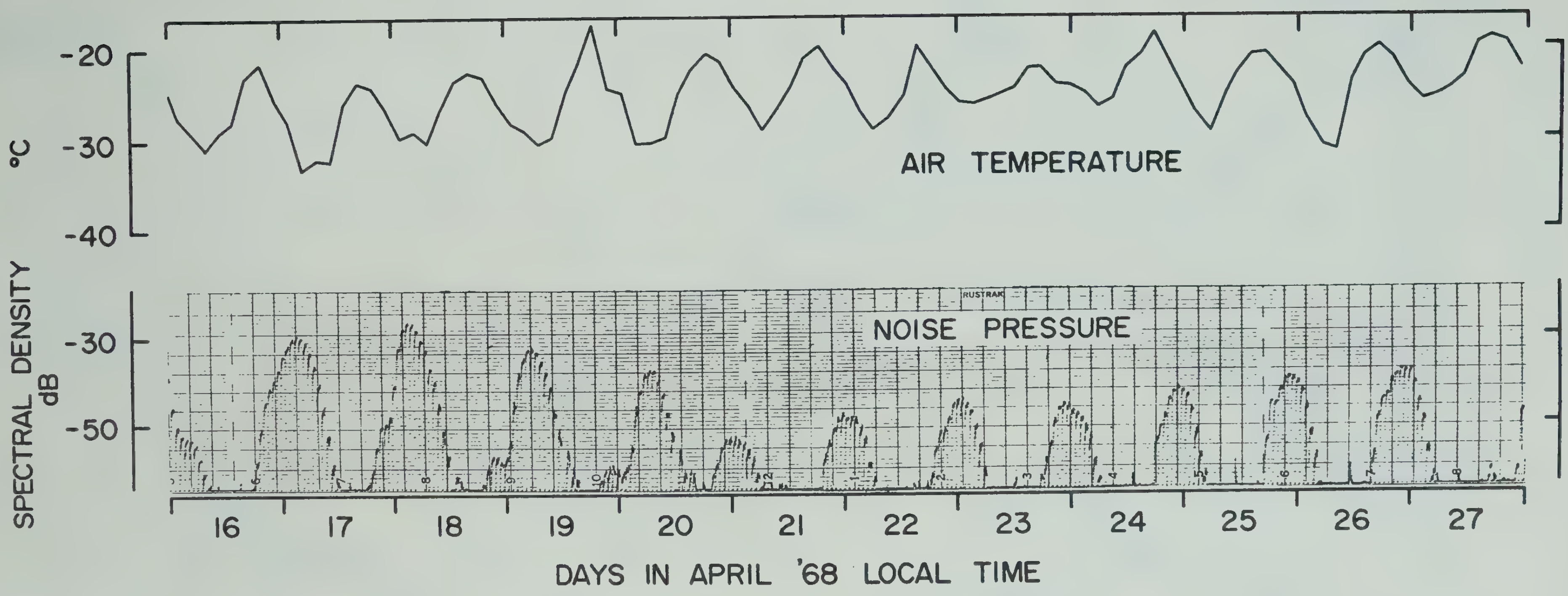

Figure 1. Diurnal variations of the noise spectral density (dB re $1(\mu b)^2$sec) in the 150-300 Hz band during April 1968. The upper curve is a plot of the air temperatures (^{O}C) at Mould Bay.

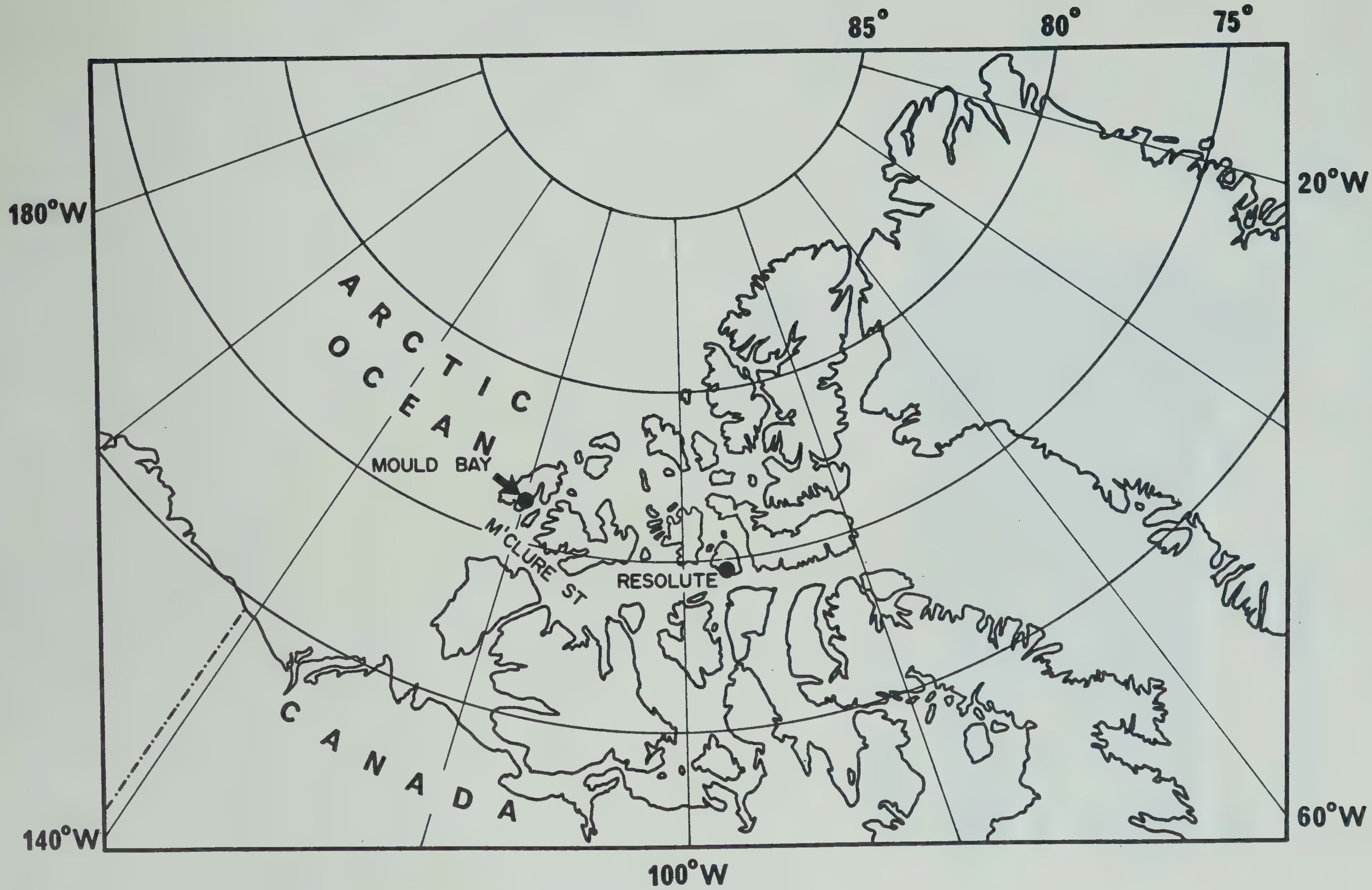

Figure 2. Canadian Arctic Archipelago.

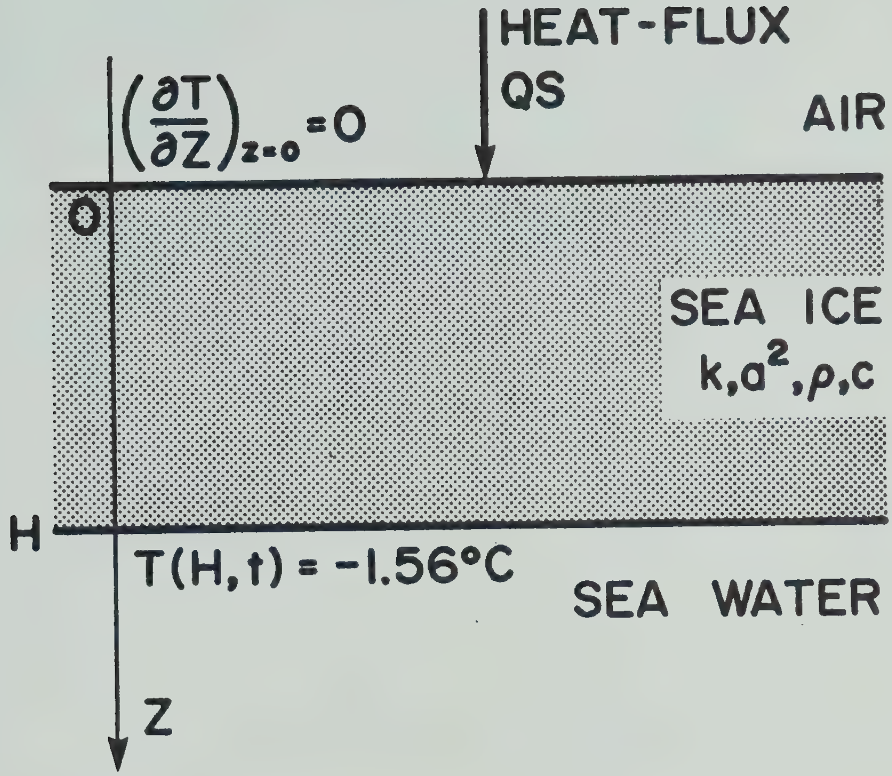

Figure 3. Geometry of the heat conduction problem.

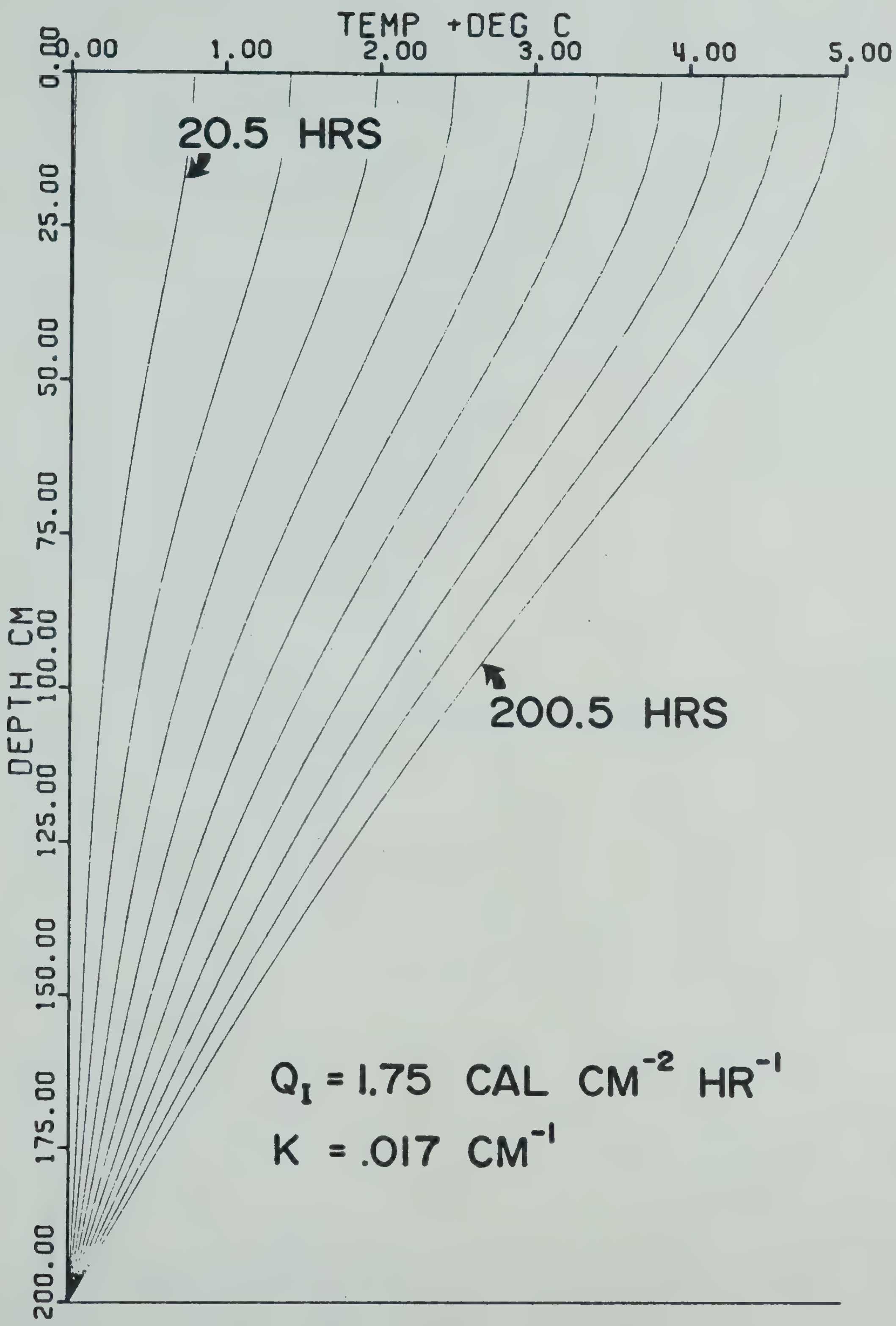

Figure 4a. $T_I(z,t)$ from equation (13), each curve being for a constant time, t, where t changes in increments of 20 hrs beginning at 0.5 hrs. Computations are for $K = 0.017\ cm^{-1}$ and $Q_I = 1.75\ cal\ cm^{-2}hr^{-1}$.

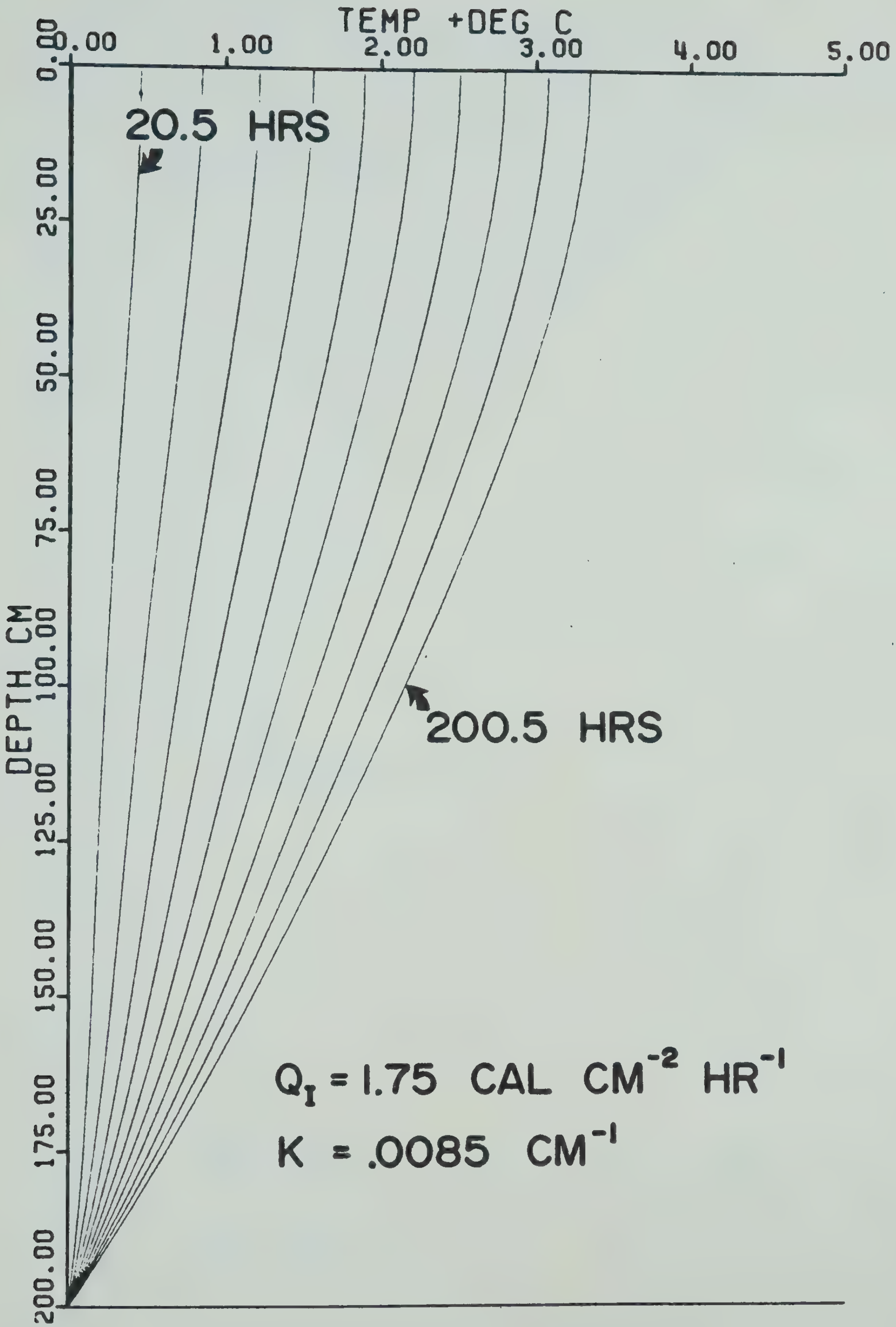

Figure 4b. $T_I(z,t)$ from equation (13), each curve being for a constant time, t, where t changes in increments of 20 hrs beginning at 0.5 hrs. Computations are for $K = 0.0085\ cm^{-1}$ and $Q_I = 1.75\ cal\ cm^{-2} hr^{-1}$.

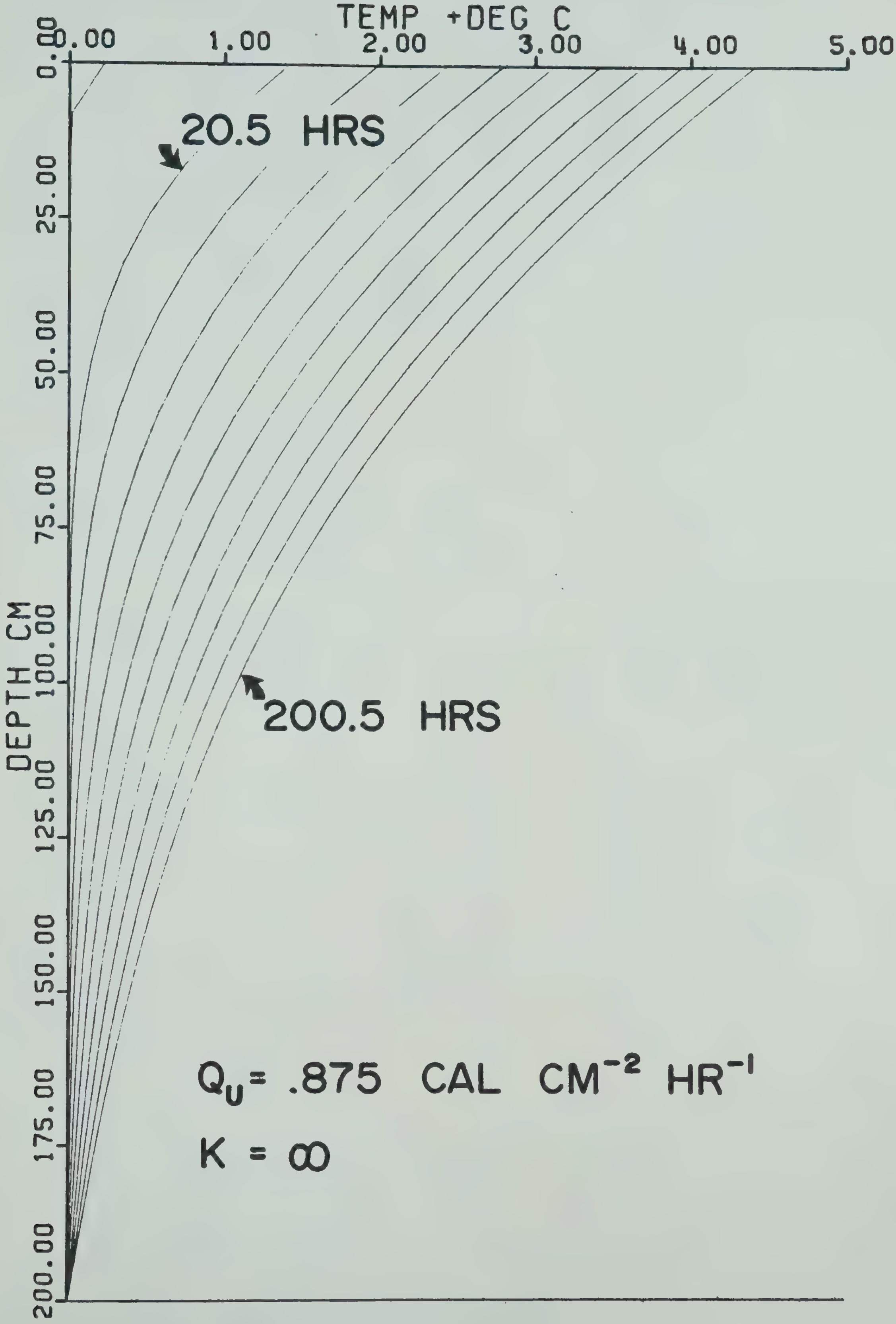

Figure 4c. $T_U(z,t)$ from equation (14), each curve being for a constant time, t, where t changes in increments of 20 hrs beginning at 0.5 hrs.

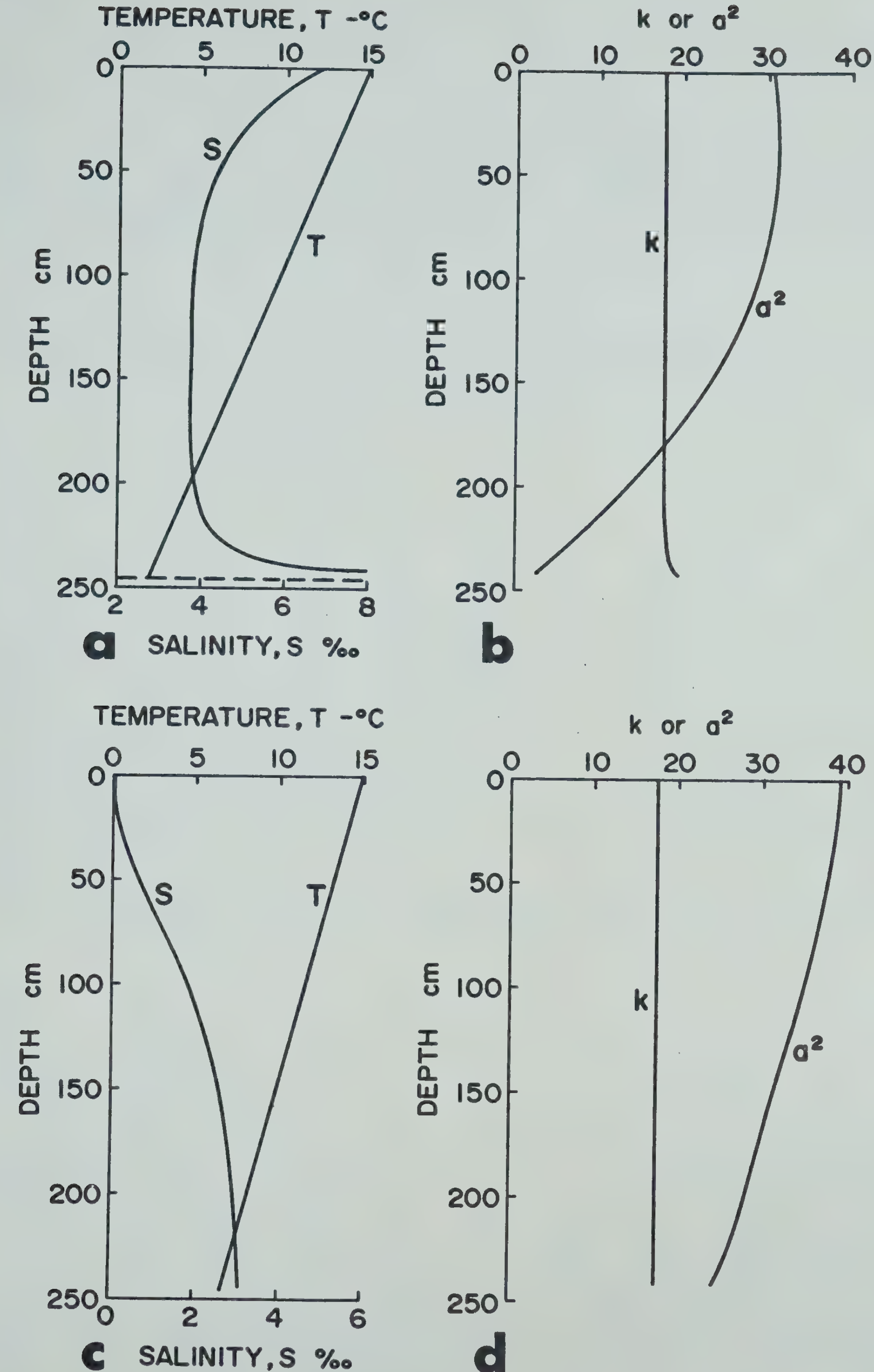

Figure 5. (a) Typical salinity (parts per thousand) and temperature (^{o}C) vs depth (cm) for winter ice in April.

(b) Using conditions for winter ice in (a), computations of the thermal diffusivity a^2 ($cm^2 hr^{-1}$) and thermal conductivity k ($cm^{-1}\ {}^{o}C^{-1} hr^{-1}$) vs depth (cm) are shown.

(c) Typical salinity (parts per thousand) and temperature (^{o}C) vs depth (cm) for polar ice in April.

(d) Using conditions for polar ice in (c), computations of the thermal diffusivity a^2 ($cm^2 hr^{-1}$) and thermal conductivity k ($cm^{-1}\ {}^{o}C^{-1} hr^{-1}$) vs depth (cm) are shown.

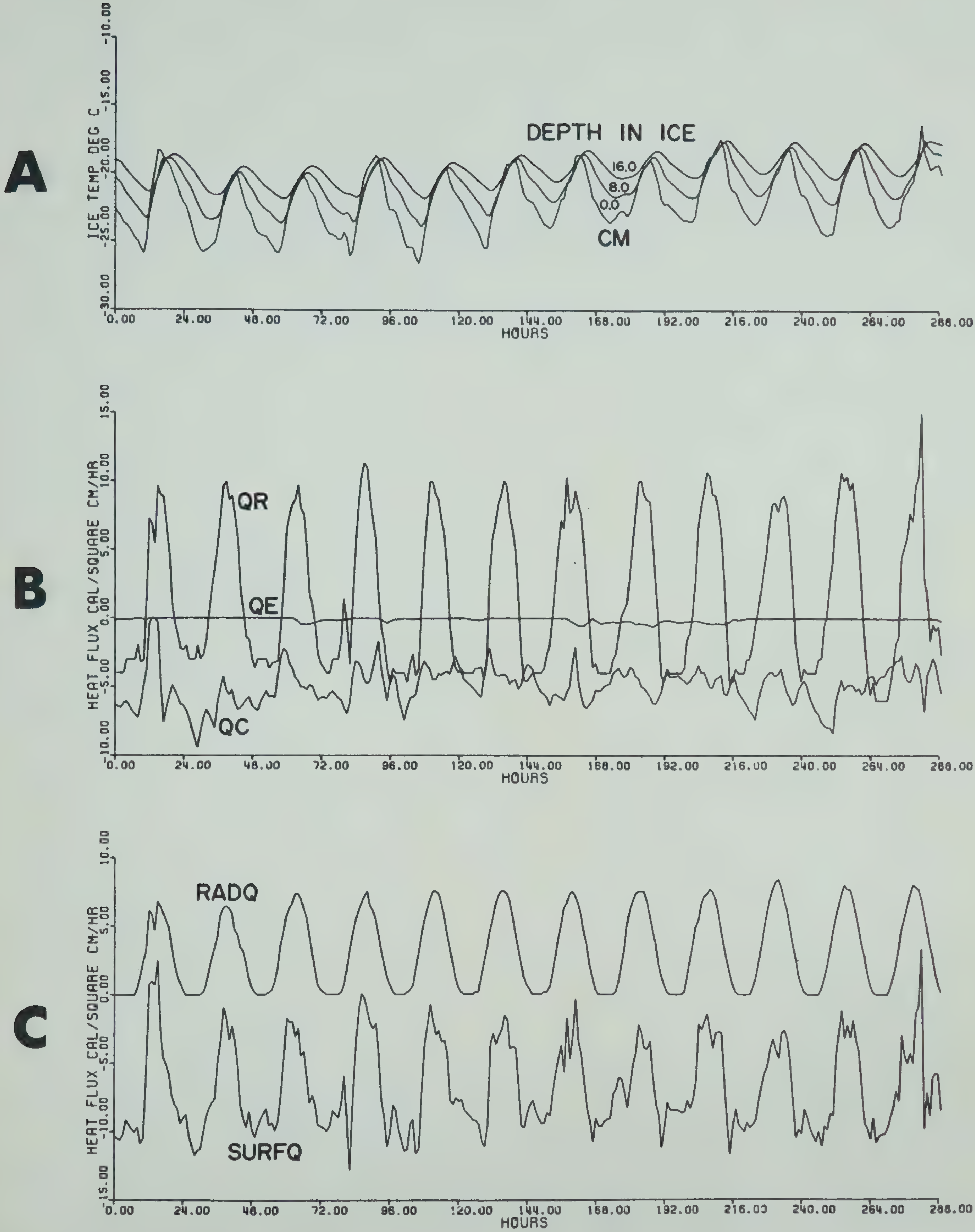

Figure 6. Computations of heat fluxes and ice temperatures for a 12-day period following 2300 hrs (LMT) on the 15th of April, 1968, using an albedo α = 0.65 and an extinction coefficient K = 0.0175 cm^{-1}.

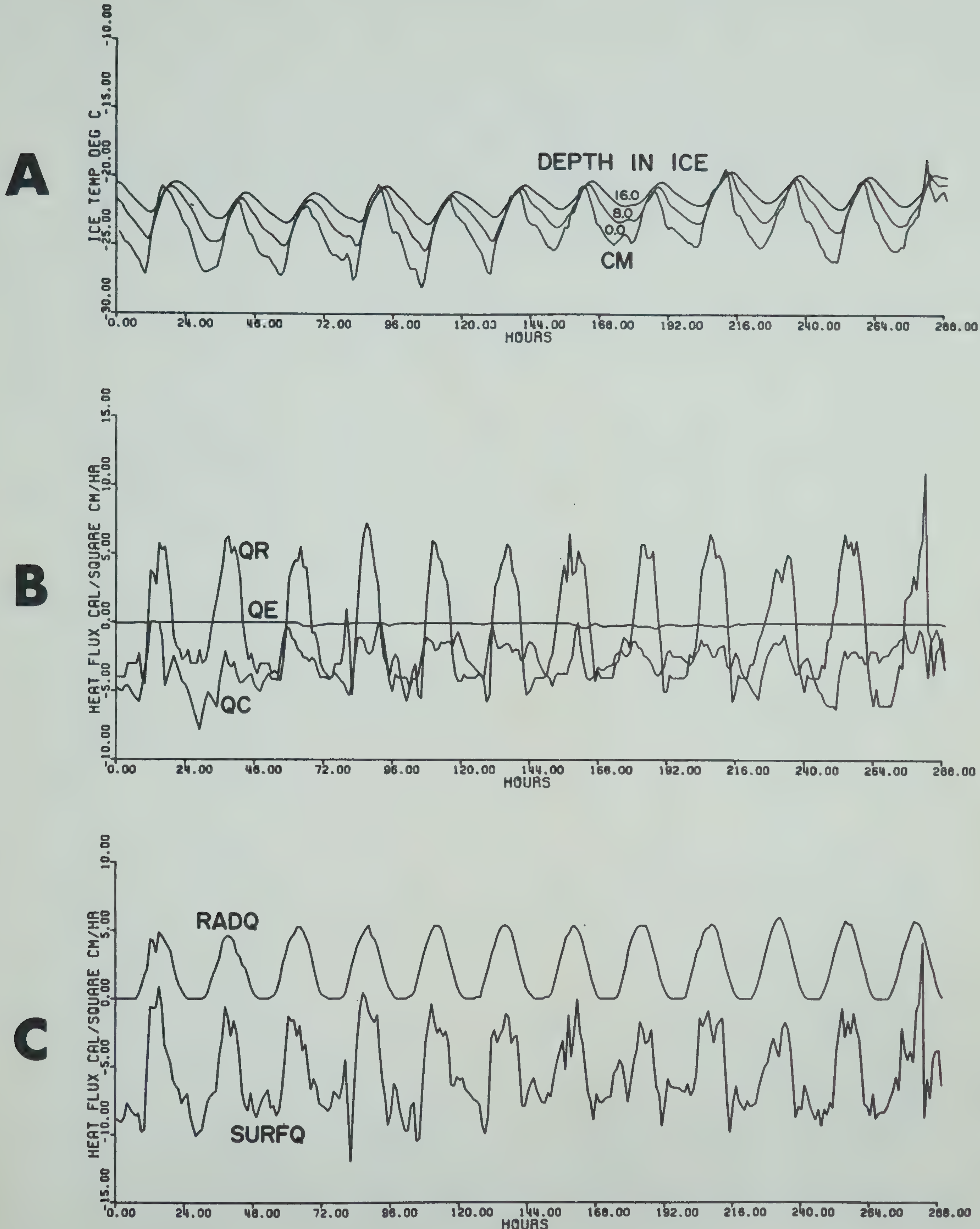

Figure 7. Computations of heat fluxes and ice temperatures for a 12-day period following 2300 hrs (LMT) on the 15th of April, 1968, using an albedo $\alpha = 0.75$ and an extinction coefficient $K = 0.034\ cm^{-1}$.

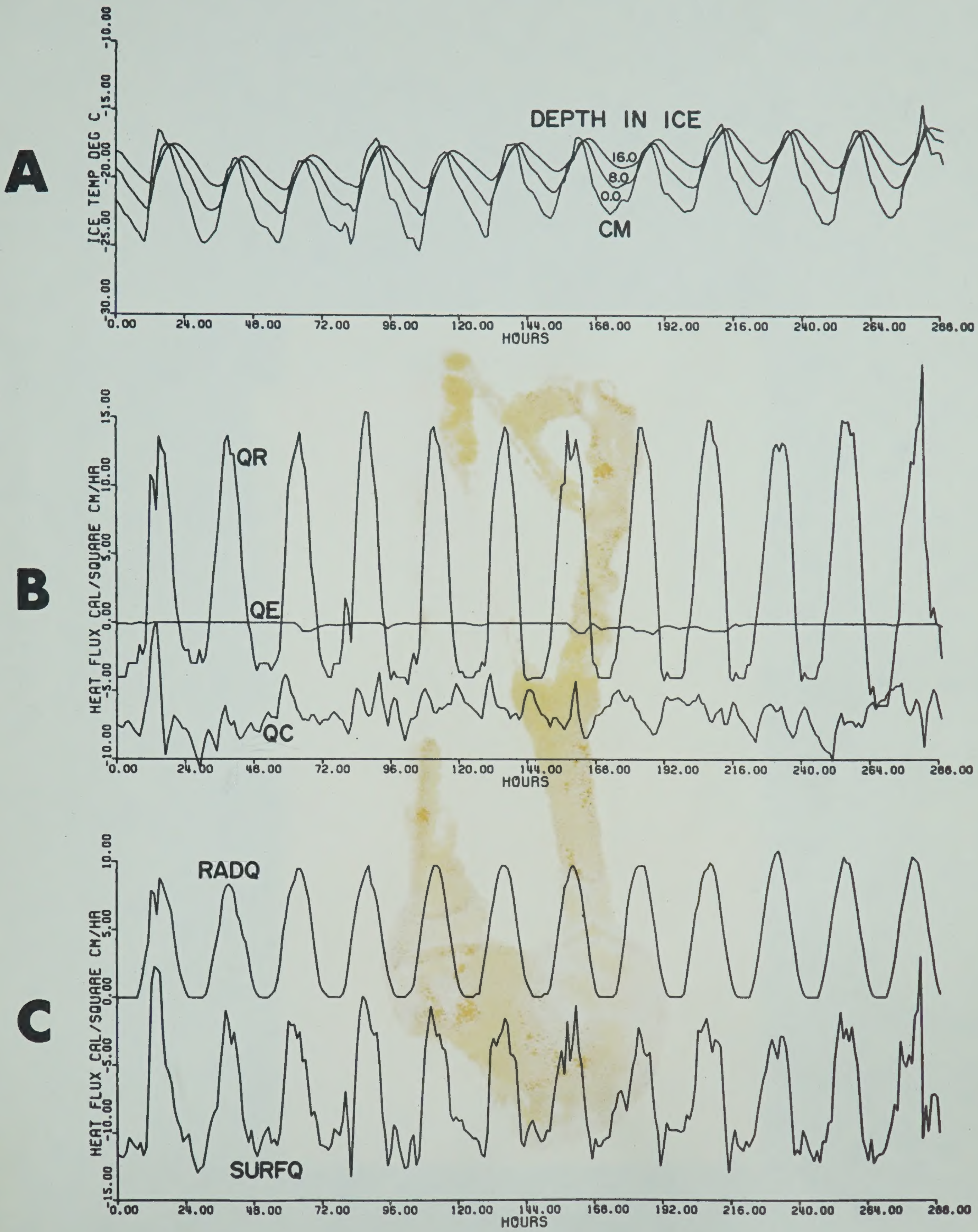

Figure 8. Computation of heat fluxes and ice temperatures for a 12-day period following 2300 hrs (LMT) on the 15th of April, 1968, using an albedo α = 0.55 and an extinction coefficient K = 0.0085 cm^{-1}.